$$+ \int_0^{Fo} \exp(\mu_i^2 v_j^2 LuFo) \int_S \varphi_k(N, Fo) \frac{\psi_i(N)}{A(N)} \mathrm{d}S \, \mathrm{d}(v_j^2 LuFo) \bigg\}. \quad (27)$$

Formulae (26) and (27) coincide completely with those of pure heat conduction if we make the substitution $\tau = v_j^2 LuFo$. Therefore nomogramm 2 given by Krischer in [5], page 8, supposing the time scale is 4 times less, becomes identical with the generally known nomogramm for plate cooling (see e.g. [6], p. 62, Fig. 29).

Consequently the decisions (24) and (25) represent linear combinations of the solution of boundary-value problems of pure heat conduction. They are easy to apply taking the numerical values for A_{kj} and B_{kj} depending on the Luikov's number, *Pn* and *Ko*Pn* given in Tables in [1].

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USE OF COATINGS OF LOW THERMAL CONDUCTIVITY TO IMPROVE FINS USED IN BOILING LIQUIDS

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NOMENCLATURE

- B, defined as $h\Delta T_2/k_1 w$;
- h, boiling heat transfer coefficient [Btu/h ft^2 °F];
- k_1, thermal conductivity of prime fin material [Btu/h ft °F];
- k_2, thermal conductivity of coating [Btu/h ft °F];
- Q, heat transfer rate [Btu/h];
- r, local radius [ft];
- r_B, radius at base of fin [ft];
- R, radius at tip of fin [ft];
- ΔT_1, temperature excess above liquid boiling point of prime fin material (a function of r) [°R];
- ΔT_2, temperature excess of wetted surface of coating (not a function of r) [°R];
- ΔT_B, temperature excess at the base of the prime fin material [°R];
- w, half-thickness of fin [ft]. (Conversion of units: 1 W = 3·414 Btu/h).

BACKGROUND

The first use of an insulating coating to improve boiling heat transfer was reported by Cowley *et al.* [1]. They showed that if the surface of a solid is so hot as to cause film boiling to an ambient liquid, one may achieve a higher heat transfer rate by applying a prescribed thickness of an insulating material. The coating thickness must be such as to result in a low enough surface temperature to cause nucleate boiling. Nucleate boiling heat transfer coefficients are often one or two orders of magnitude greater than film boiling coefficients, thus the gain in surface coefficient may offset the penalty of thermal resistance of the coating. Cowley tried coatings of varnish, vaseline, asbestos, sodium silicate, and other materials on five metals in three liquids and obtained improvements in the heat flux by as much as 500 per cent.

Recently, Rubin *et al.* [2] rediscovered the use of insulation to improve boiling heat fluxes. They reported tests with a single copper spine, 0·79 in. long by 0·4 in. dia., sheathed in stainless steel. The thickness of the stainless steel coating varied from 0·18 in. at the fin-base to zero at the tip, calculated so as to cause the peak heat transfer coefficient for boiling to occur simultaneously everywhere on the fin. The improvement in heat duty is unknown, because no tests were reported for the uncoated spine.

The idea of using a coating material of low thermal conductivity to improve fins in boiling liquids is attractive. In principle the uncoated fins could be of any geometry. In the following, we present a simple mathematical analysis for transverse circular fins, originally of constant thickness. One such finned-tube was constructed of copper coated with bismuth, and tested in Freon-113, $CCl_2F\text{-}CClF_2$, as described.

MATHEMATICAL PROCEDURE

For the composite fins considered here, it is essential that the thermal conductivity of the prime fin material be higher than that of the coating. The analysis is based on the assumption that the heat flow is one dimensional, radially, in the prime fin material and also one dimensional (at right angles) across the coating. Rubin used these same assumptions. It is desired that the entire exposed surface of the coating be isothermal, and the coating thickness is selected to satisfy this condition. For a given tube temperature, a single heat transfer coefficient will describe all positions on the fin surface. This is in sharp contrast to uncoated fins for which huge variations in h may occur [3].

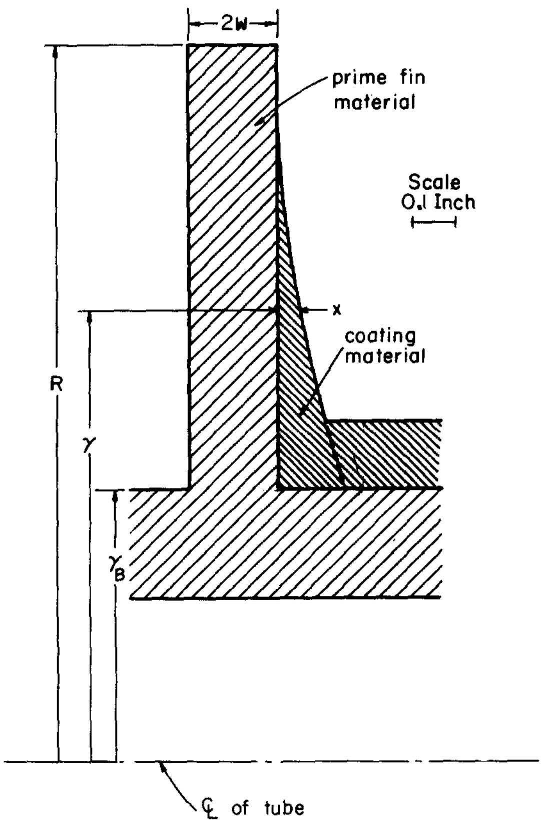

FIG. 1. Sketch of fin with coating on one side. The relative dimensions here are those of the fin-tube in Figs. 3 and 4.

Figure 1 shows a cross section of one fin and the coating on one side. At position r the heat conducted in the prime fin material is given by equation (1).

$$Q = -k_1(2\pi r \,.\, 2w)\,\mathrm{d}(\Delta T_1)/\mathrm{d}r. \tag{1}$$

The heat transferred to the ambient fluid in distance $\mathrm{d}r$ is given by equation (2).

$$-\mathrm{d}Q = h(\Delta T_2)4\pi r\,\mathrm{d}r. \tag{2}$$

Letting $B = h\Delta T_2/k_1 w$, the governing expression becomes equation (3) with the boundary conditions as shown.

$$\frac{\mathrm{d}^2(\Delta T_1)}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}(\Delta T_1)}{\mathrm{d}r} = B. \tag{3}$$

$$At\ r = R, \Delta T_1 = \Delta T_2 \text{ and } \frac{\mathrm{d}(\Delta T_1)}{\mathrm{d}r} = \frac{-h\Delta T_2}{k_1} = -Bw$$

The solution of equation (3) is equation (4).

$$\Delta T_1 = \Delta T_2 + B[(wR + \tfrac{1}{2}R^2)\ln(R/r) - \tfrac{1}{4}(R^2 - r^2)]. \tag{4}$$

The local heat flux across the insulation equals the local heat rejection rate to the liquid. These are equated, and the insulation thickness x is found, equation (5).

$$x = k_2(\Delta T_1 - \Delta T_2)/h\Delta T_2. \tag{5}$$

Substituting equation (4) into equation (5), the desired expression relating x and r is obtained.

$$\frac{xwk_1}{R^2k_2} = \left(\frac{w}{R} + \frac{1}{2}\right)\ln\left(\frac{R}{r}\right) - \frac{1}{4}\left(1 - \frac{r^2}{R^2}\right). \tag{6}$$

Equation (6) is graphed in Fig. 2. For a particular fin, the ordinate will apply for values of r/R from r_B/R to 1. The total heat duty of one fin is given by equation (7).

$$Q = 2\pi(R^2 - r_B{}^2 + 2wR)h\Delta T_2. \tag{7}$$

EXPERIMENTAL

The finned tube used was Fin A previously used by Bondurant and Westwater [4]. It was of copper with integral fins 1 in. high, 0·2 in. thick, 0·76 in. clearance, $r_B = 0{\cdot}626$ in. and a tube wall thickness of 0·25 in. The tube wall temperature was determined from average readings of three thermocouples in deep holes in the tube wall drilled parallel to the geometric axis.

Bismuth, Baker and Adamson 99·8 per cent purity, was selected as the coating material for reasons of convenience. It has a thermal conductivity of 3·9 Btu/h ft °F (at 210°F), which is about 2 per cent of the value for copper, thus it

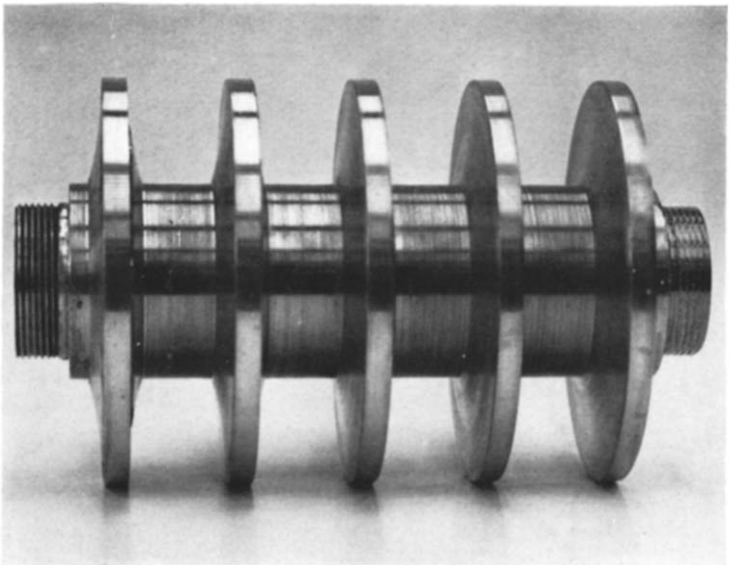

Fig. 3. Finned tube of copper coated with bismuth. The fin on the left is viewed edge-on, and the tapered bismuth coating is evident.

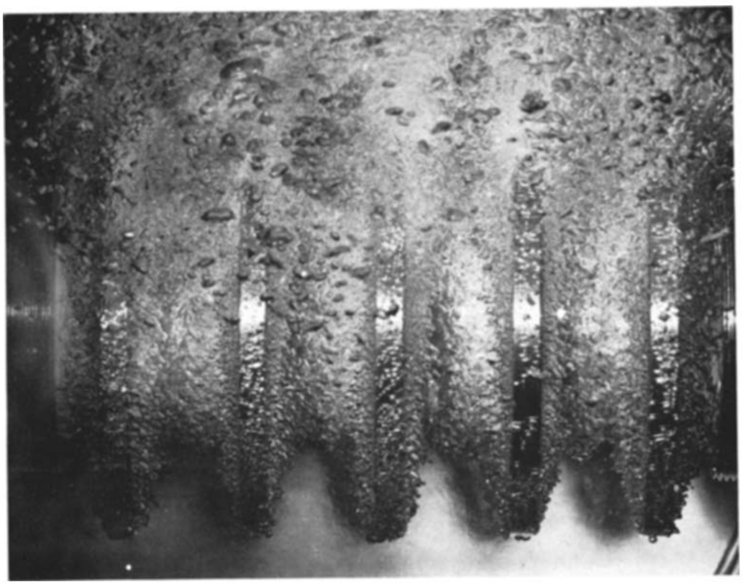

Fig. 4. Freon-113 at 14 300 Btu/h. Nucleate boiling occurs everywhere on the surface. The surface ΔT is 31·5°F everywhere, whereas the fin base has a ΔT of 112°F.

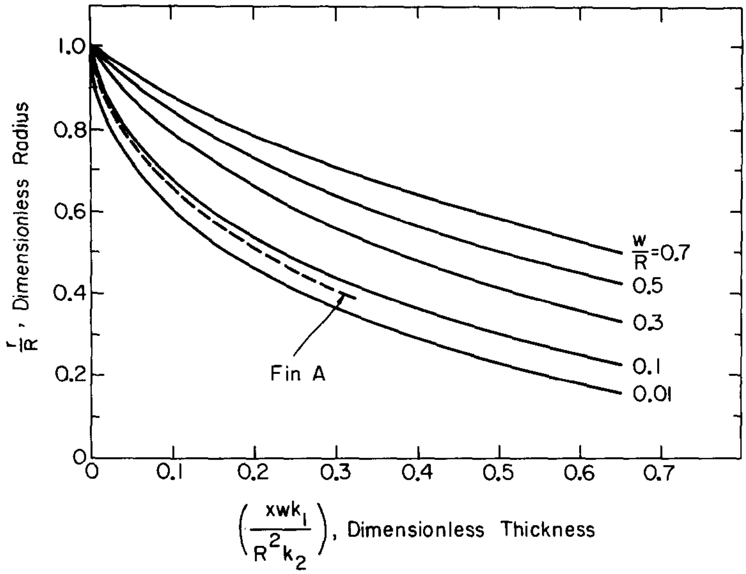

FIG. 2. Design chart for the insulating coating thickness, equation (6).

required dimensions which could be achieved readily on a lathe. It has a melting point of 520°F and can be cast on copper with reasonable ease. The preheated copper finned tube was immersed in liquid bismuth and then allowed to cool. It was necessary that both metals be very clean and free of oxygen; an atmosphere of nitrogen was present throughout the heating, casting, and cooling operations. The copper and bismuth assembly was then machined to the desired shape drawn to scale in Fig. 1 and illustrated in Fig. 3. The maximum thickness of the bismuth was 0·153 in. The final surface preparation was a manual polish with 3/0 emery paper.

The tests were carried out in equipment described in detail previously [4]. The coated, finned tube was mounted horizontally in a 2·5 gal stainless-steel boiler having glass windows. Vapor rose to overhead condensers having 13·3 ft^2. The tube was heated internally by a helical carbon ribbon, electrically heated, in a nitrogen atmosphere. Heat duties were measured by three methods: the electrical power input, the boil-up rate, and the condenser water heat pickup. These agreed within 10 per cent. The Freon-113 was boiled under a vacuum during start-up to remove dissolved air. Data were taken at atmospheric pressure. Eleven successful runs were made at successively higher heat duties. When the maximum heat duty was reached, the sudden temperature excursion caused part of the bismuth to melt. This terminated the investigation.

RESULTS

The point in using an insulating coating on the fins is to achieve an isothermal surface. Figure 4 is a photograph of Freon-113 boiling on the bismuth-coated copper fins. The visual appearance of the bubbles on the surface in the lower half of the picture is remarkably uniform. No large

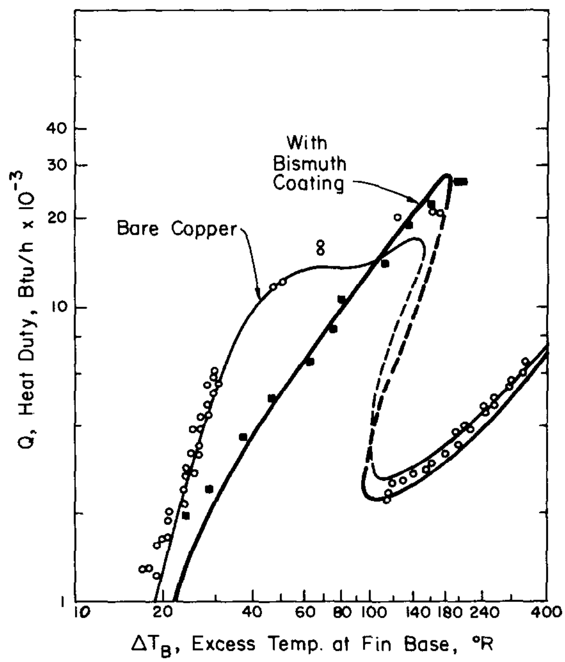

FIG. 5. Experimental and predicted performance for the finned tube, before and after coating with bismuth. The smooth curves are predicted; the symbols are experimental. The uncoated fin information is from [4].

film-boiling bubbles are seen. This contrasts with photographs [4] of the uncoated tube in use. Figure 4 indicates success in obtaining a uniform surface temperature and surface heat transfer coefficient.

The experimental data for the finned tube, both before and after coating, are given in Fig. 5. The measured improvement in the peak heat duty was 29 per cent. The predicted performances are given by the curves; they indicate a possible improvement of 68 per cent. The discrepancy may result from imperfect machining of the bismuth or theoretical errors arising from the assumption of one-dimensional conduction. However, the agreement between the predicted and measured performance for the coated fins is close. The predicted critical temperature difference is 185°F for the coated fins as contrasted with 140°F for the uncoated fins. The respective measured values were 200 and 169°F The coated fins show no superiority at low values of ΔT. Their attraction is the increased heat duty at high values of ΔT. It is significant that the coating geometry is independent of the heat flux and the choice of liquid. The type of coating, the durability of the coating, the way to apply the coating, the surface texture, and the wettability of the coating are significant problems which need further study.

ACKNOWLEDGEMENT

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